Limits and Derivatives

- **Derivatives**
  - Suppose f is a real-valued function and a is a point in its domain of definition. The derivative of f at a [denoted by $f'(a)$] is defined as

    $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, provided the limit exists.
    Derivative of $f(x)$ at a is denoted by $f'(a)$.

  - Suppose f is a real-valued function. The derivative of $f\left\{\text{denoted by } f'(x) \text{ or } \frac{d}{dx}[f(x)]\right\}$ is defined as

    $\frac{d}{dx}[f(x)] = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, provided the limit exists.
    This definition of derivative is called the first principle of derivative.

    Example: Find the derivative of $f(x) = x^2 + 2x$ using first principle of derivative.

    Solution: We know that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

    $\therefore f'(x) = \lim_{h \to 0} \frac{(x+h) + 2(x+h) - (x^2 + 2x)}{h}$

    $= \lim_{h \to 0} \frac{x^2 + h^2 + 2hx + 2x + 2h - x^2 - 2x}{h}$

    $= \lim_{h \to 0} \frac{h^2 + 2hx + 2h}{h}$

    $= \lim_{h \to 0} (h + 2x + 2)$

    $= 0 + 2x + 2 = 2x + 2$

    $f'(x) = 2x + 2$

- **Derivatives of Polynomial Functions**

  For the functions u and v (provided u' and v' are defined in a common domain),

  - $(u \pm v)' = u' \pm v'$
  - $(uv)' = u'v + uv'$ (Product rule)
  - $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ (Quotient rule)

- **Derivatives of Trigonometric Functions**

- $\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$ for any positive integer *n*
- $\frac{d}{dx}\left(a_{n}x^{n} + a_{n-1}x^{n-1} + \ldots + a_{1}x + a_{0}\right) = na_{n}x^{n-1} + \left(n-1\right)a_{n-1}x^{n-1} + \ldots + a_{1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^{2} x$

Example: Find the derivative of the function $f(x) = \left(3x^{2} + 4x + 1\right) \cdot \tan x$

Solution: We have**,**
f(x)=(3x2+4x+1).tan xDifferentiating both sides with respect to x, f'(x)=(3x2+4x+1).ddx(tan x)+tan x .ddx(3x2+4x+1)f'(x)=(3x2+4x+1).(sec2x)+tan x(6x+4)f'(x)=(3x2+4x+1).(sec2x)+(6x+4)tan x